



《多模态机器学习》

第六章 多模态对齐

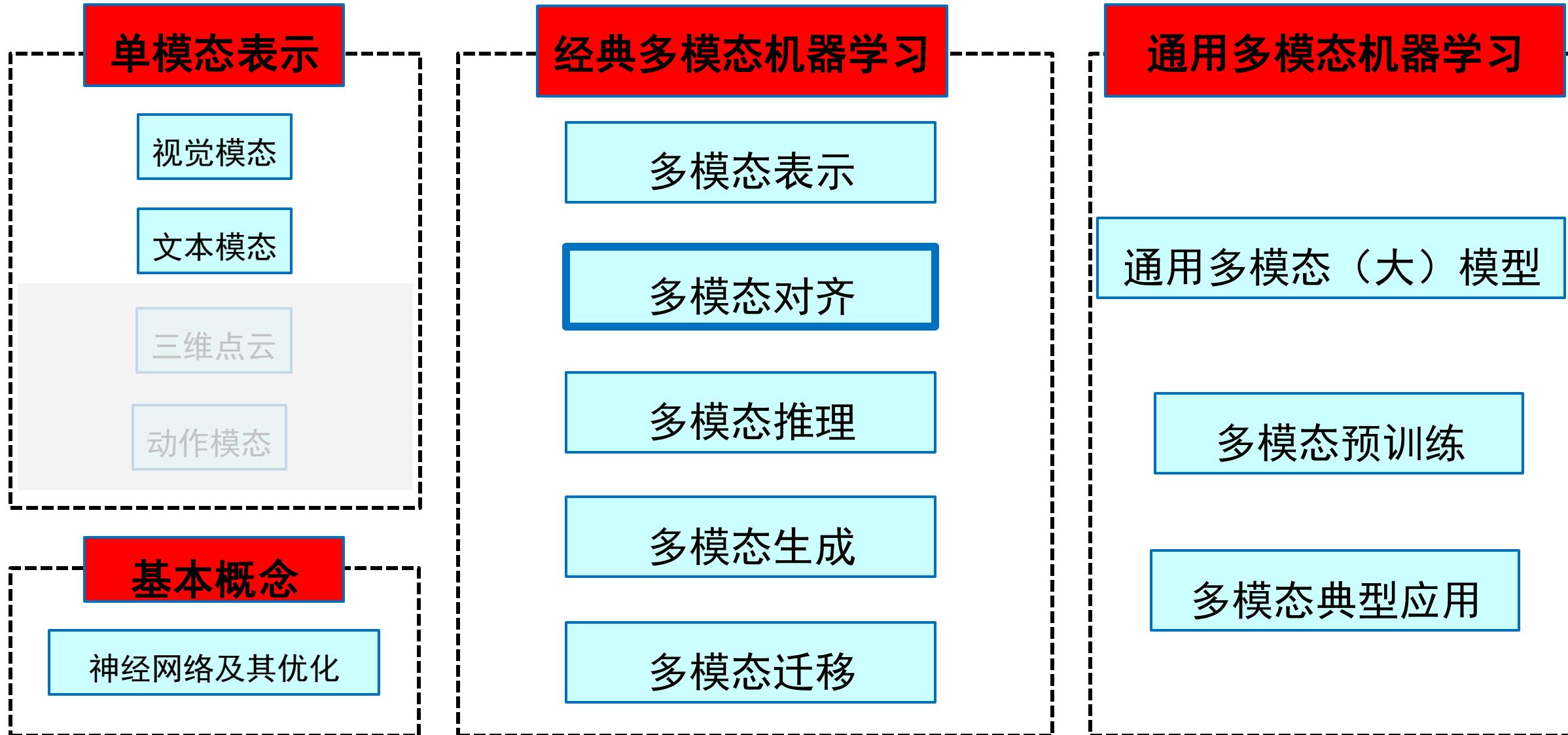
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2024年秋季

课程提纲

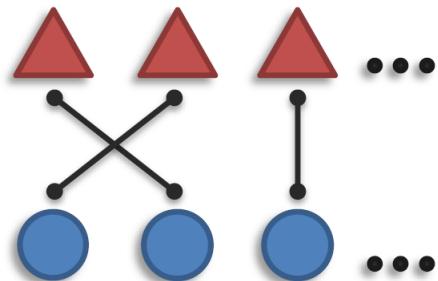


Task 2: Alignment

Definition: Identifying and modeling cross-modal connections between all elements of multiple modalities, building from the data structure

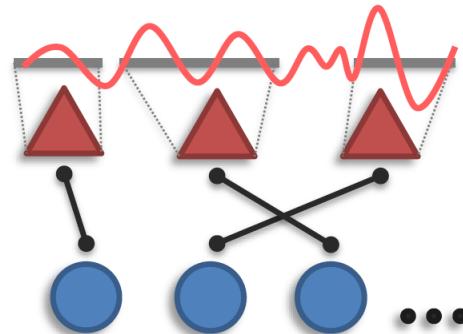
Sub-challenges:

Discrete Alignment



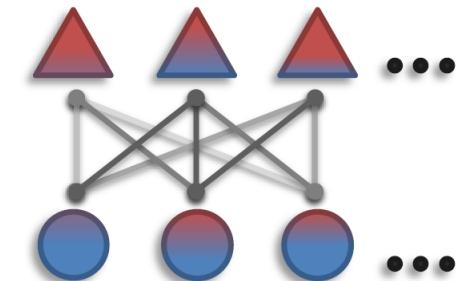
Discrete elements
and connections

Continuous Alignment



Segmentation and
continuous warping

Contextualized Representation



Alignment + representation

内容提纲

① Discrete alignment

① Local alignment

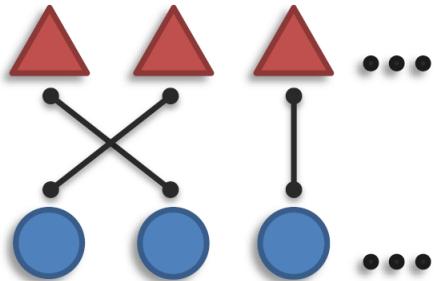
② Global alignment

② Continuous alignment

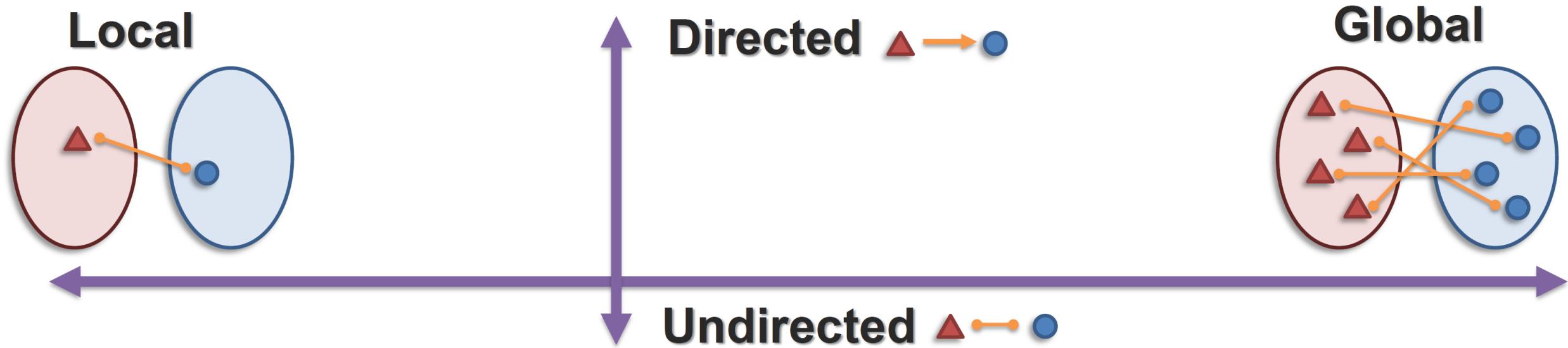
① Continuous warping

② Discretization and segmentation

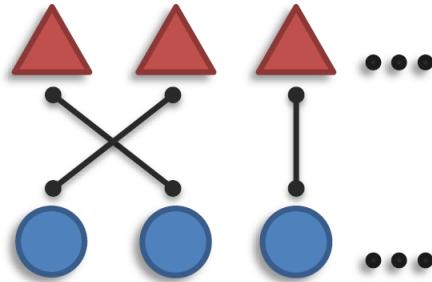
Sub-Challenge 2a: Discrete Alignment



Definition: Identify and model connections between elements of multiple modalities



Connections



Why should 2 elements be connected?

Statistical



Association

Dependency



e.g., correlation,
co-occurrence



e.g., causal,
temporal

Semantic



Correspondence

Relationship



e.g., grounding



e.g., function

Language Grounding

Definition: Tying language (words, phrases,...) to non-linguistic elements, such as the visual world (objects, people, ...)



A **woman** reading **newspaper**

Statistical



Association



e.g., correlation,
co-occurrence

Dependency



e.g., causal,
temporal

Semantic



Correspondence



e.g., grounding

Relationship

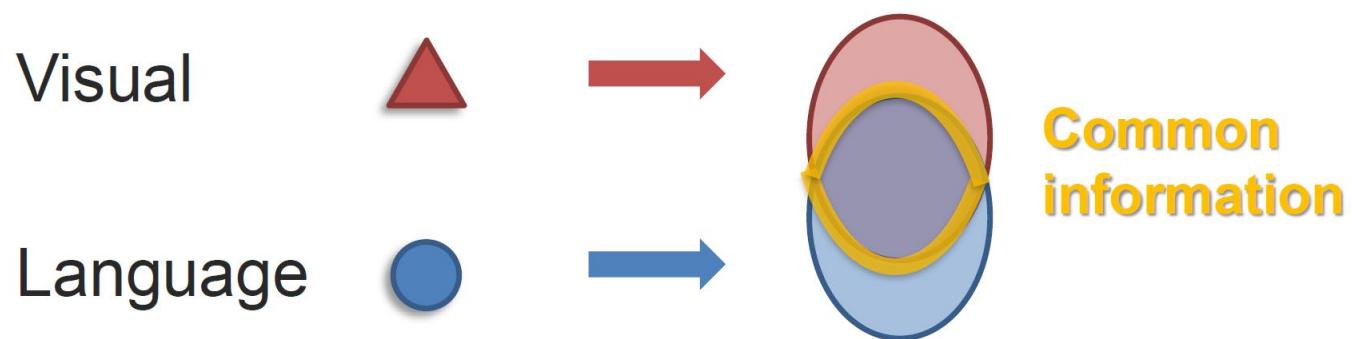


e.g., function



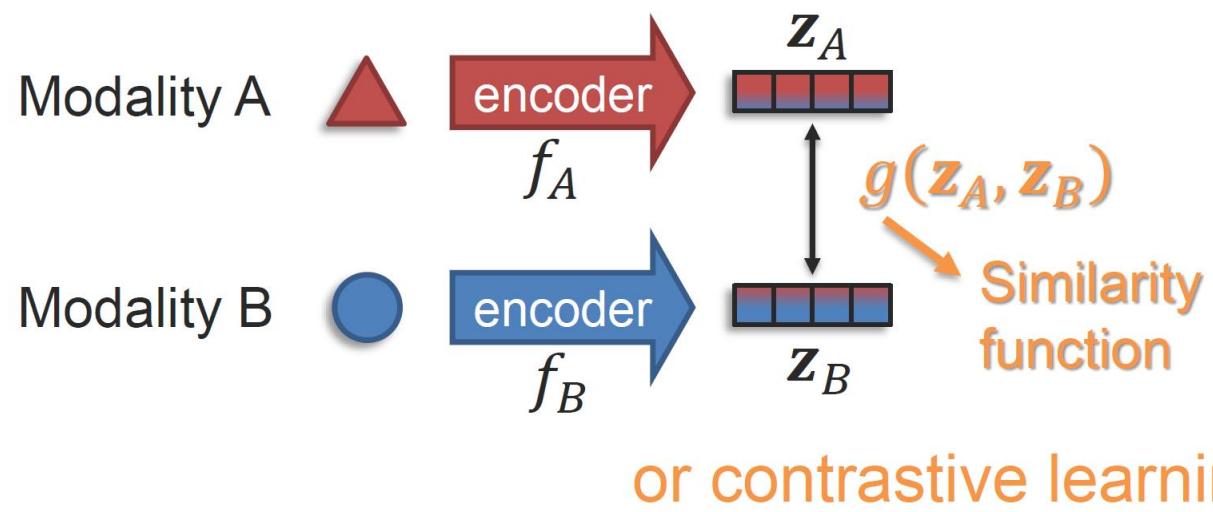
e.g., grounding

Local Alignment – Coordinated Representations

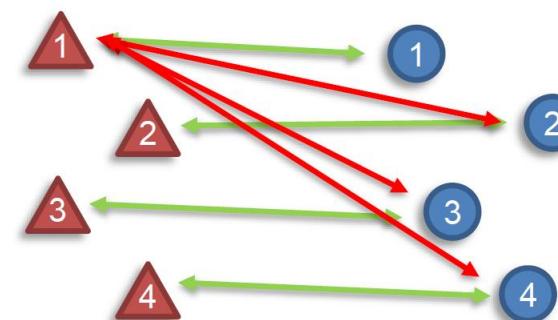


A woman reading newspaper

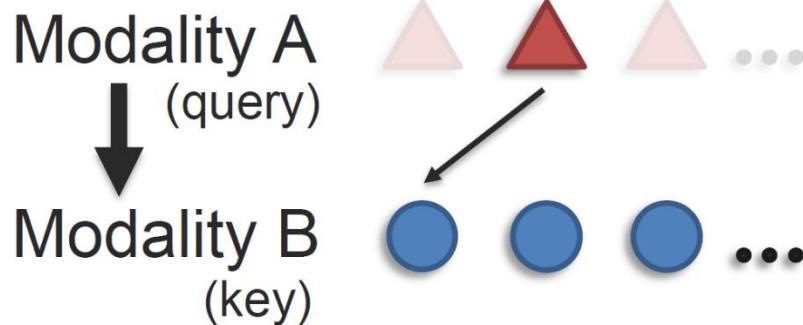
Learning coordinated representations:



Supervision: Paired data



Directed Alignment

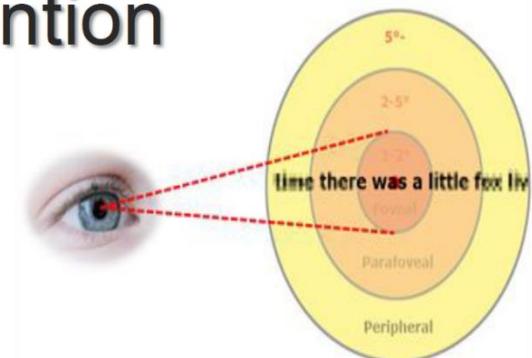


A woman is throwing a frisbee

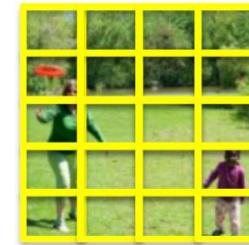
Which object?

A red curved arrow originates from the text 'Which object?' and points to the red frisbee in the photograph.

Attention



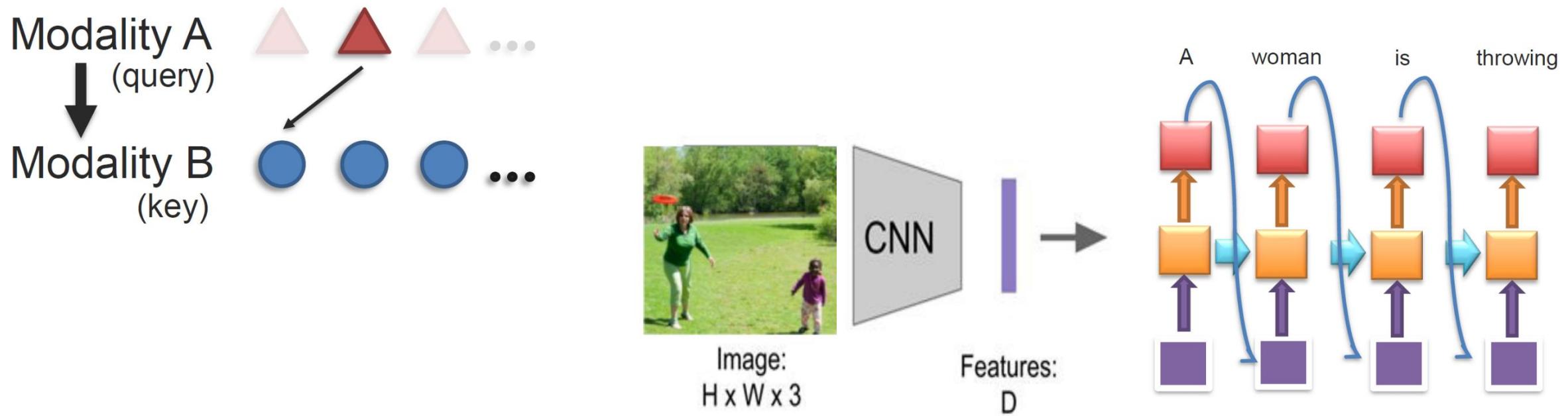
1 Soft attention



2 Hard attention

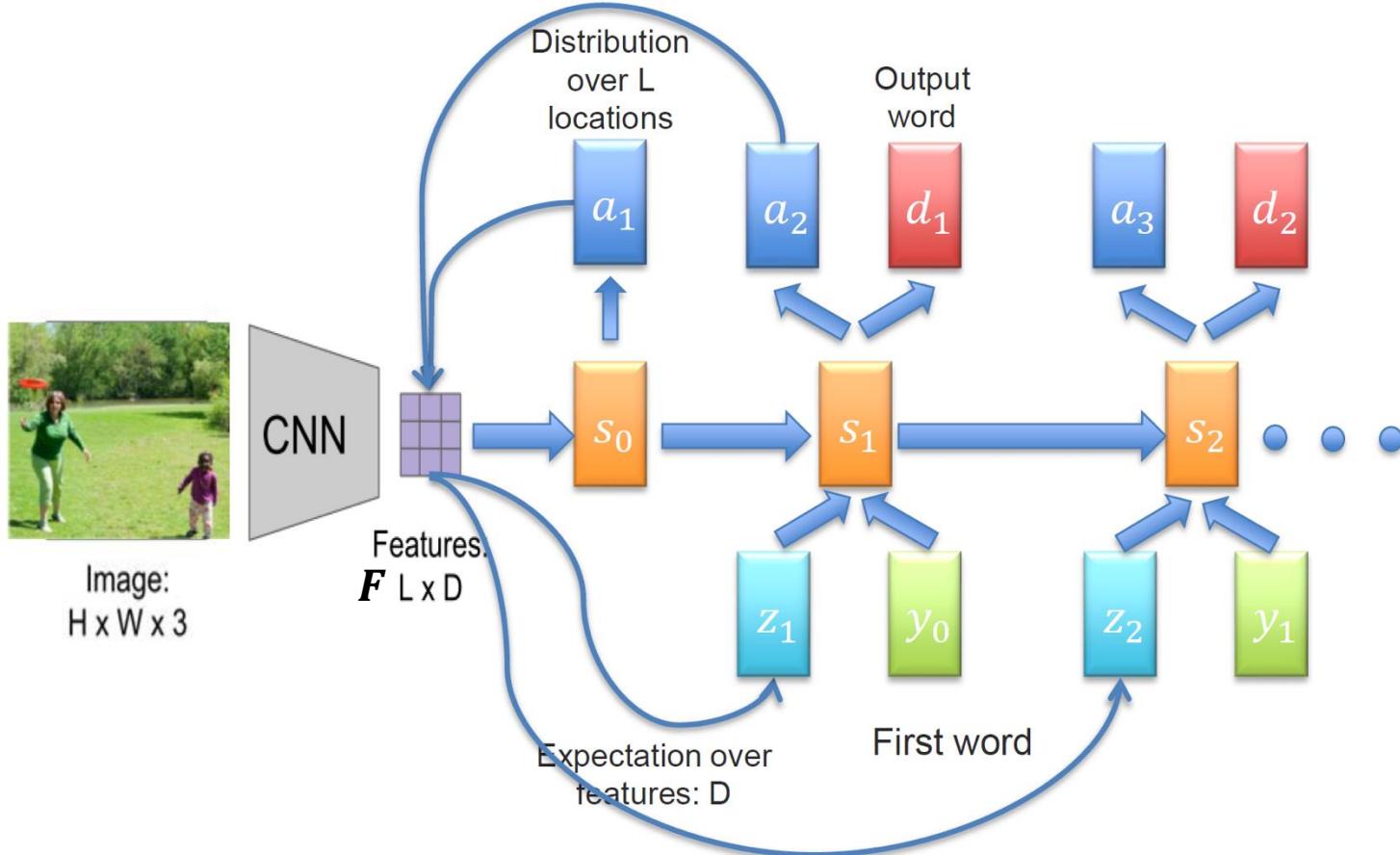


Cross-modal Interactions



Should we always use the final layer of the CNN for all generated words?

Directed Alignment – Image Captioning



Attention Gates

Before:

$$p(y_i|y_1, \dots, y_{i-1}, \mathbf{x}) = g(y_{i-1}, \mathbf{s}_i, \mathbf{z}),$$

where $\mathbf{z} = \mathbf{h}_T$, last encoder state and \mathbf{s}_i is the current state of the decoder

Now:

$$p(y_i|y_1, \dots, y_{i-1}, \mathbf{x}) = g(y_{i-1}, \mathbf{s}_i, \mathbf{z}_i)$$

Have an attention “gate”

- A different context \mathbf{z}_i used at each time step!
- $\mathbf{z}_i = \sum_{j=1}^L \alpha_{ij} \mathbf{f}_j$ α_{ij} is the (scalar) attention for word i at image position j

Attention Gates

So how do we determine a_{ij} ?

$$a_{ij} = \frac{\exp(e_{ij})}{\sum_{k=1}^T \exp(e_{ik})} \quad \Rightarrow \text{softmax, making sure they sum to 1}$$

where:

$$e_{ij} = \mathbf{v}^\top \sigma(\mathbf{W}\mathbf{s}_{i-1} + \mathbf{U}\mathbf{f}_j)$$

a feedforward network that can tell us how important the current encoding is

$\mathbf{v}, \mathbf{W}, \mathbf{U}$ – learnable weights

$$\mathbf{z}_i = \sum_{j=1}^L a_{ij} \mathbf{f}_j$$

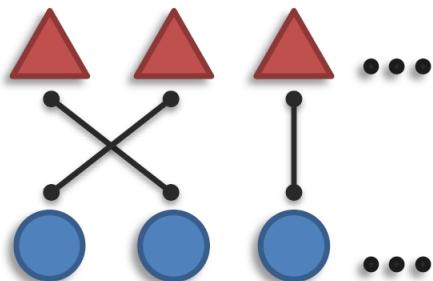
expectation of the context (a fancy way to say it's a weighted average)

Example – Image Captioning



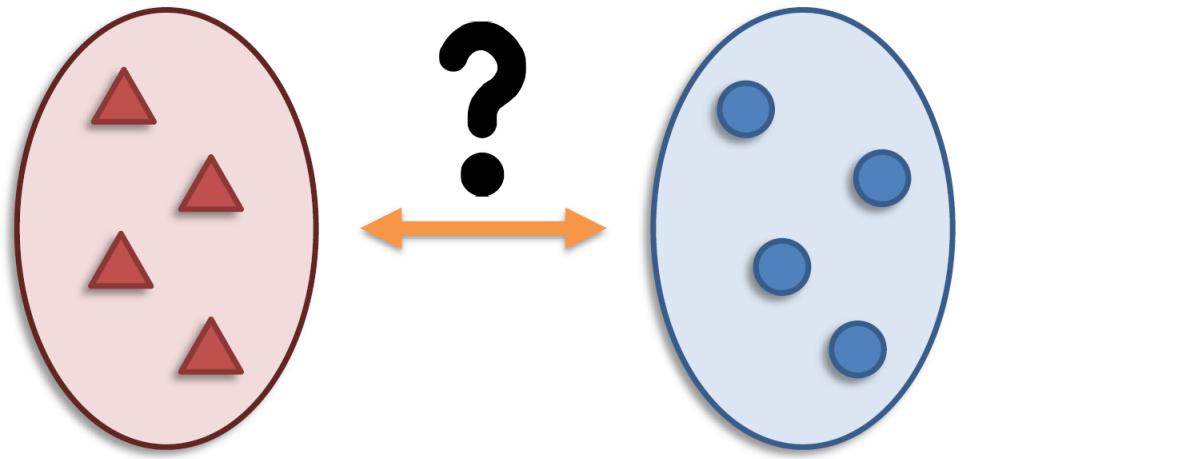
Global Alignment

Visual



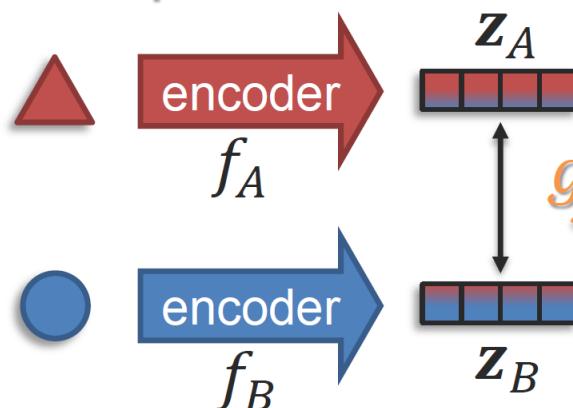
Language

Latent pairing information

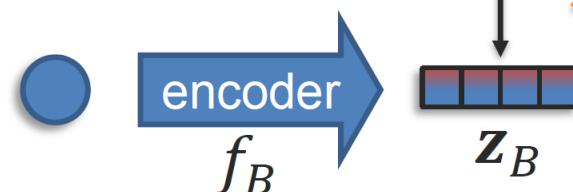


Jointly optimize representation + global alignment:

Modality A

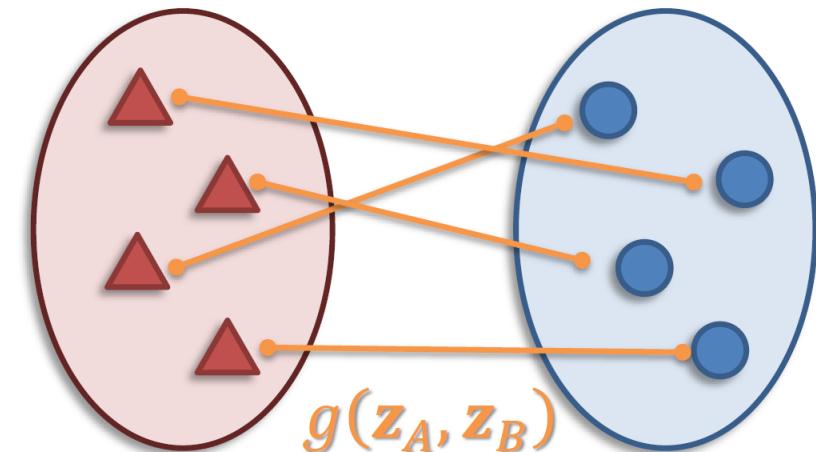
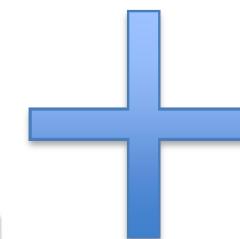


Modality B



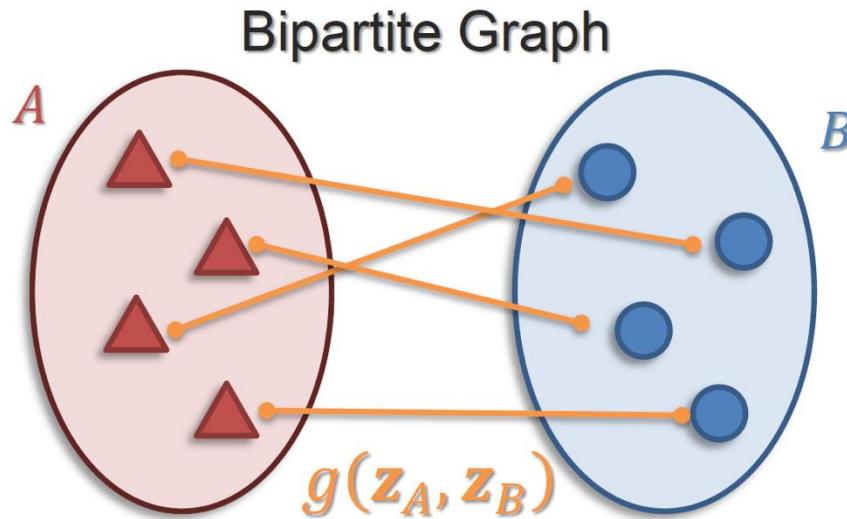
(representation)

$g(\mathbf{z}_A, \mathbf{z}_B)$
Coordination
function



(global alignment)

Global Alignment



Assignment:
(vector of indices)

$$f: A \rightarrow B$$

Similarity weights:

$$w_{i,f(i)} = g(\mathbf{z}_A^i, \mathbf{z}_B^{f(i)})$$

Maximize:

$$\max_{f \in S_n} \sum_{i=1}^N w_{i,f(i)}$$

Initial assumptions:

- Same number of elements in A and B modalities
- 1-to-1 “hard” alignment between elements
- All elements assigned (aka “perfect matching”)

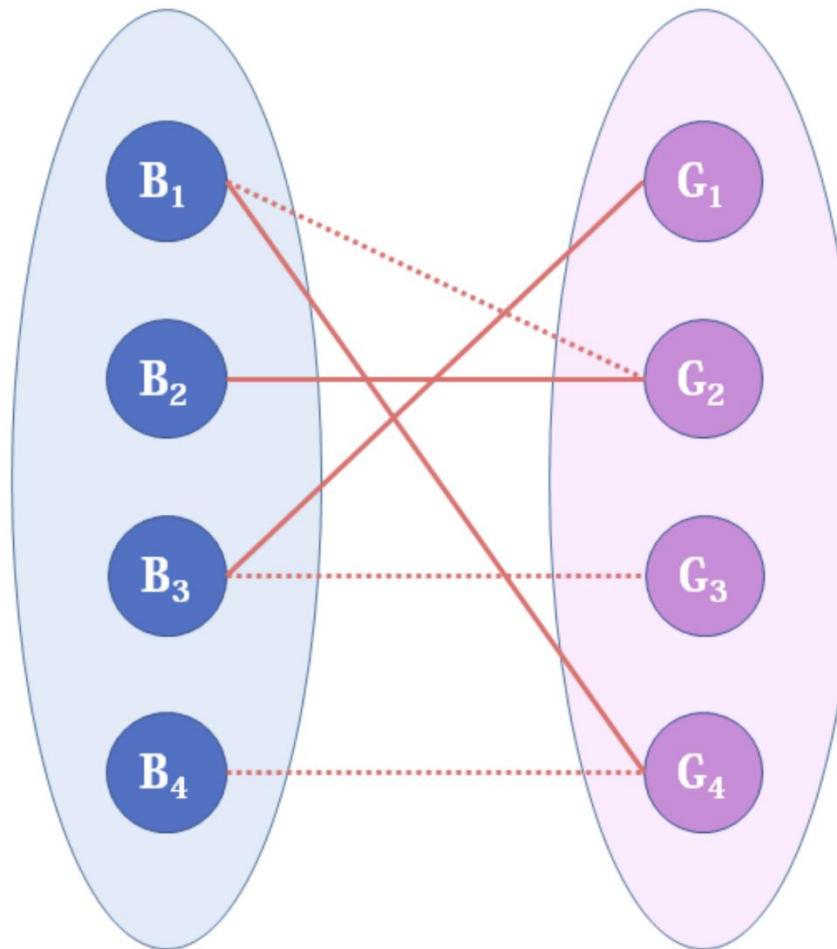
→ How to solve?

Naive solution: check all assignments

Hungarian algorithm

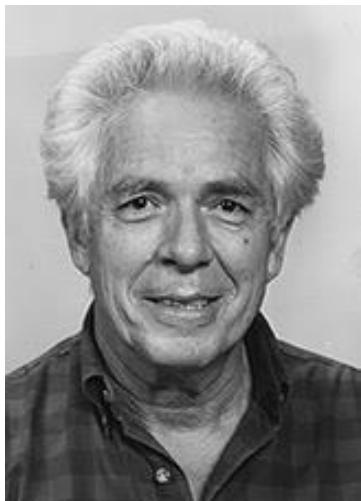
“红娘算法”

Boys Girls



Hungarian algorithm

The Hungarian method, known also as the Kuhn–Munkres algorithm or Munkres assignment algorithm



Harold William Kuhn
(July 29, 1925 – July 2, 2014)

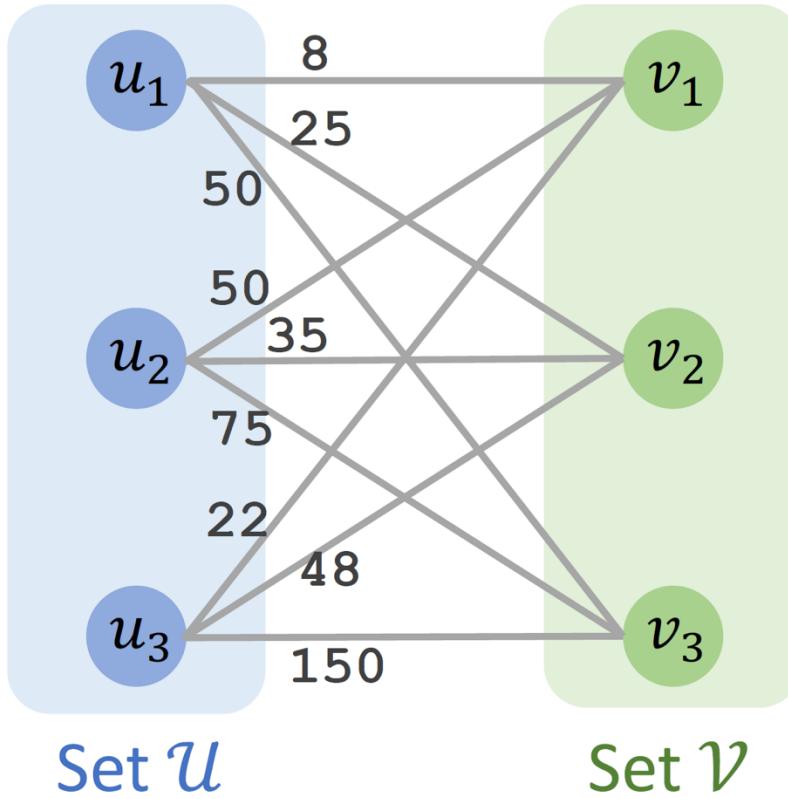


Dénes König
(September 21, 1884 – October 19, 1944)



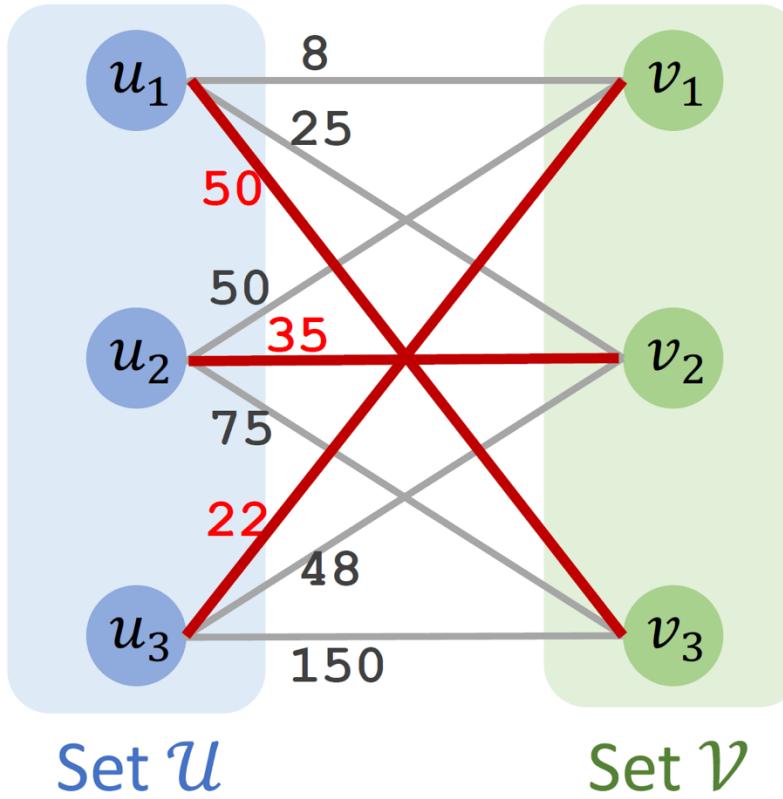
Jenő Elek Egerváry
(April 16, 1891 – November 30, 1958)

Minimum-Weight Bipartite Matching



	v_1	v_2	v_3
u_1	8	25	50
u_2	50	35	75
u_3	22	48	150

Minimum-Weight Bipartite Matching



	v_1	v_2	v_3
u_1	8	25	50
u_2	50	35	75
u_3	22	48	150

The minimum sum of weight is $50 + 35 + 22 = 107$.

Minimum-Weight Bipartite Matching

Subtract Row Minima

	v_1	v_2	v_3
u_1	8	25	50
u_2	50	35	75
u_3	22	48	150

Minimum-Weight Bipartite Matching

Subtract Row Minima

	v_1	v_2	v_3
u_1	8	25	50
u_2	50	35	75
u_3	22	48	150

Minimum-Weight Bipartite Matching

Subtract Row Minima

	v_1	v_2	v_3
u_1	8 -8	25 -8	50 -8
u_2	50 -35	35 -35	75 -35
u_3	22 -22	48 -22	150 -22

Minimum-Weight Bipartite Matching

Subtract Row Minima

Now, the row minima are zeros.

	v_1	v_2	v_3
u_1	0	17	42
u_2	15	0	40
u_3	0	26	128

Minimum-Weight Bipartite Matching

Subtract Column Minima

	v_1	v_2	v_3
u_1	0	17	42
u_2	15	0	40
u_3	0	26	128

Minimum-Weight Bipartite Matching

Subtract Column Minima

	v_1	v_2	v_3
u_1	0	17	42
u_2	15	0	40
u_3	0	26	128

Minimum-Weight Bipartite Matching

Subtract Column Minima

	v_1	v_2	v_3
u_1	0 -0	17 -0	42 -40
u_2	15 -0	0 -0	40 -40
u_3	0 -0	26 -0	128 -40

Minimum-Weight Bipartite Matching

Subtract Column Minima

Now, the column minima are zeros.

	v_1	v_2	v_3
u_1	0	17	2
u_2	15	0	0
u_3	0	26	88

Minimum-Weight Bipartite Matching

Iteration 1A

Repeat the followings:

- A. Cover all the zeros with a minimum number of lines.
- B. Decide whether to stop.
- C. Create additional zeros.

	v_1	v_2	v_3
u_1	0	17	2
u_2	15	0	0
u_3	0	26	88

Minimum-Weight Bipartite Matching

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	v_1	v_2	v_3
u_1	0	17	2
u_2	15	0	0
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Minimum-Weight Bipartite Matching

Iteration 1A

Repeat the followings:

- A. Cover all the zeros with a minimum number of lines.
- B. Decide whether to stop.
- C. Create additional zeros.

	v_1	v_2	v_3
u_1	0	17	2
u_2	15	0	0
u_3	0	26	88

Not optimal!

Minimum-Weight Bipartite Matching

Iteration 1A

Repeat the followings:

- A. Cover all the zeros with a minimum number of lines.
- B. Decide whether to stop.
- C. Create additional zeros.

	v_1	v_2	v_3
u_1	0	17	2
u_2	15	0	0
u_3	0	26	88

Minimum-Weight Bipartite Matching

Iteration 1B

Repeat the followings:

A. Cover all the zeros with a minimum number of lines.

→ B. Decide whether to stop.

C. Create additional zeros.

- If n lines are required, the algorithm stops.
- If less than n lines are required, then continue with Step C.

	v_1	v_2	v_3
u_1	0	17	2
u_2	15	0	0
u_3	0	26	88

Minimum-Weight Bipartite Matching

Iteration 1B

Repeat the followings:

- A. Cover all the zeros with a minimum number of lines.
- B. Decide whether to stop.
- C. Create additional zeros.

First, find the smallest element (denote k) that is not covered by a line.

	v_1	v_2	v_3
u_1	0	17	2
u_2	15	0	0
u_3	0	26	88

Minimum-Weight Bipartite Matching

Iteration 1C

Repeat the followings:

- A. Cover all the zeros with a minimum number of lines.
- B. Decide whether to stop.
- C. Create additional zeros.

First, find the smallest element (denote k) that is not covered by a line.

	v_1	v_2	v_3
u_1	0	17	2 $=k$
u_2	15	0	0
u_3	0	26	88

Minimum-Weight Bipartite Matching

Iteration 1C

Repeat the followings:

- A. Cover all the zeros with a minimum number of lines.
- B. Decide whether to stop.
- C. Create additional zeros.

Second, subtract k from all uncovered elements.

	v_1	v_2	v_3
u_1	0	17 -2	2 -2
u_2	15	0	0
u_3	0	26 -2	88 -2

Minimum-Weight Bipartite Matching

Iteration 1C

Repeat the followings:

- A. Cover all the zeros with a minimum number of lines.
- B. Decide whether to stop.
- C. Create additional zeros.

Second, subtract k from all uncovered elements.

	v_1	v_2	v_3
u_1	0	15	0
u_2	15	0	0
u_3	0	24	86

Minimum-Weight Bipartite Matching

Iteration 1C

Repeat the followings:

- A. Cover all the zeros with a minimum number of lines.
- B. Decide whether to stop.
- C. Create additional zeros.

Third, add k to all the elements that are covered twice.

	v_1	v_2	v_3
u_1	0	15	0
u_2	15	0	0
u_3	0	24	86

Minimum-Weight Bipartite Matching

Iteration 1C

Repeat the followings:

- A. Cover all the zeros with a minimum number of lines.
- B. Decide whether to stop.
- C. Create additional zeros.

Second, subtract k from all uncovered elements.

	v_1	v_2	v_3
u_1	0	15	0
u_2	15	0	0
u_3	0	24	86

Minimum-Weight Bipartite Matching

Iteration 1C

Repeat the followings:

- A. Cover all the zeros with a minimum number of lines.
- B. Decide whether to stop.
- C. Create additional zeros.

Third, add k to all the elements that are covered twice.

	v_1	v_2	v_3
u_1	0	15	0
u_2	15	0	0
u_3	0	24	86

Minimum-Weight Bipartite Matching

Iteration 1C

Repeat the followings:

- A. Cover all the zeros with a minimum number of lines.
- B. Decide whether to stop.
- C. Create additional zeros.

Third, add k to all the elements that are covered twice.

	v_1	v_2	v_3
u_1	0	15	0
u_2	17	0	0
u_3	0	24	86

Minimum-Weight Bipartite Matching

Iteration 2

Repeat the followings:

- A. Cover all the zeros with a minimum number of lines.
- B. Decide whether to stop.
- C. Create additional zeros.

	v_1	v_2	v_3
u_1	0	15	0
u_2	17	0	0
u_3	0	24	86

Minimum-Weight Bipartite Matching

Iteration 2A

Repeat the followings:

- A. Cover all the zeros with a minimum number of lines.
- B. Decide whether to stop.
- C. Create additional zeros.

	v_1	v_2	v_3
u_1	0	15	0
u_2	17	0	0
u_3	0	24	86

Minimum-Weight Bipartite Matching

Iteration 2A

Repeat the followings:

- A. Cover all the zeros with a minimum number of lines.
- B. Decide whether to stop.
- C. Create additional zeros.

	v_1	v_2	v_3
u_1	0	15	0
u_2	17	0	0
u_3	0	24	86

At least 3 lines are needed.

Minimum-Weight Bipartite Matching

Iteration 2B

Repeat the followings:

- A. Cover all the zeros with a minimum number of lines.
- B. Decide whether to stop.**
- C. Create additional zeros.

If n lines are required, the algorithm stops.

The algorithm stops.

	v_1	v_2	v_3
u_1	0	15	0
u_2	17	0	0
u_3	0	24	86

Minimum-Weight Bipartite Matching

Output the matching

	v_1	v_2	v_3
u_1	0	15	0
u_2	17	0	0
u_3	0	24	86

Minimum-Weight Bipartite Matching

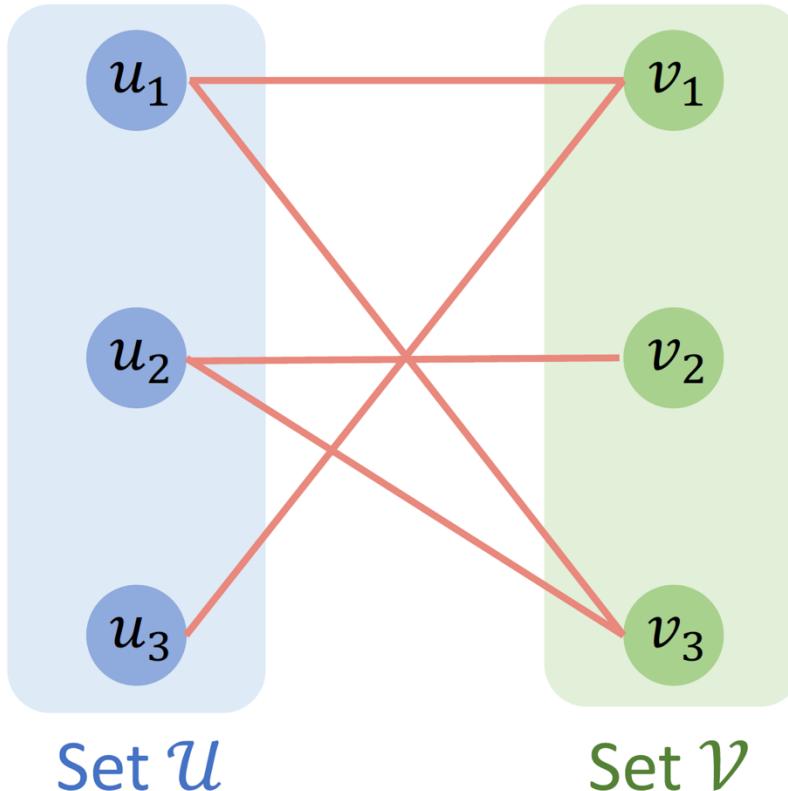
Output the matching

	v_1	v_2	v_3
u_1	0	15	0
u_2	17	0	0
u_3	0	24	86

- Choose a matching among the **zeros**.
- Think of the **zeros** as **edges**.

Minimum-Weight Bipartite Matching

Output the matching

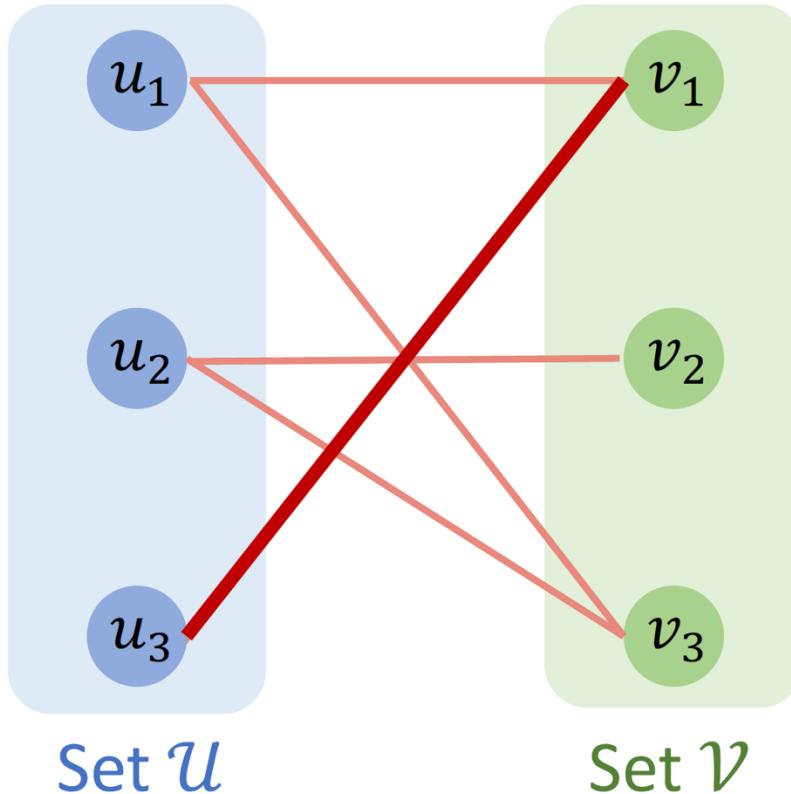


	v_1	v_2	v_3
u_1	0	15	0
u_2	17	0	0
u_3	0	24	86

- Choose a matching among the **zeros**.
- Think of the **zeros** as **edges**.

Minimum-Weight Bipartite Matching

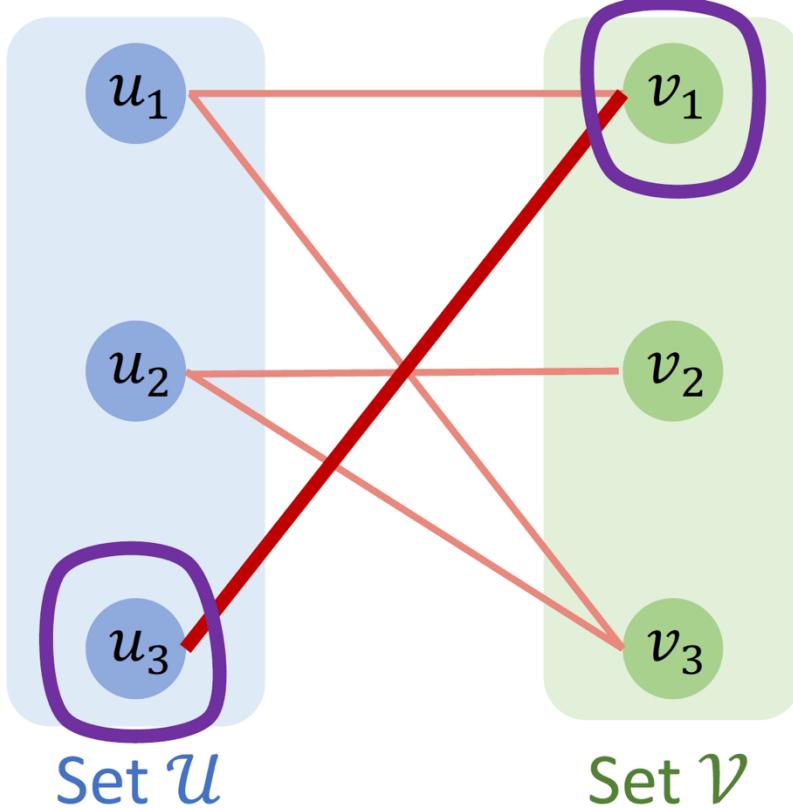
Output the matching



	v_1	v_2	v_3
u_1	0	15	0
u_2	17	0	0
u_3	0	24	86

Minimum-Weight Bipartite Matching

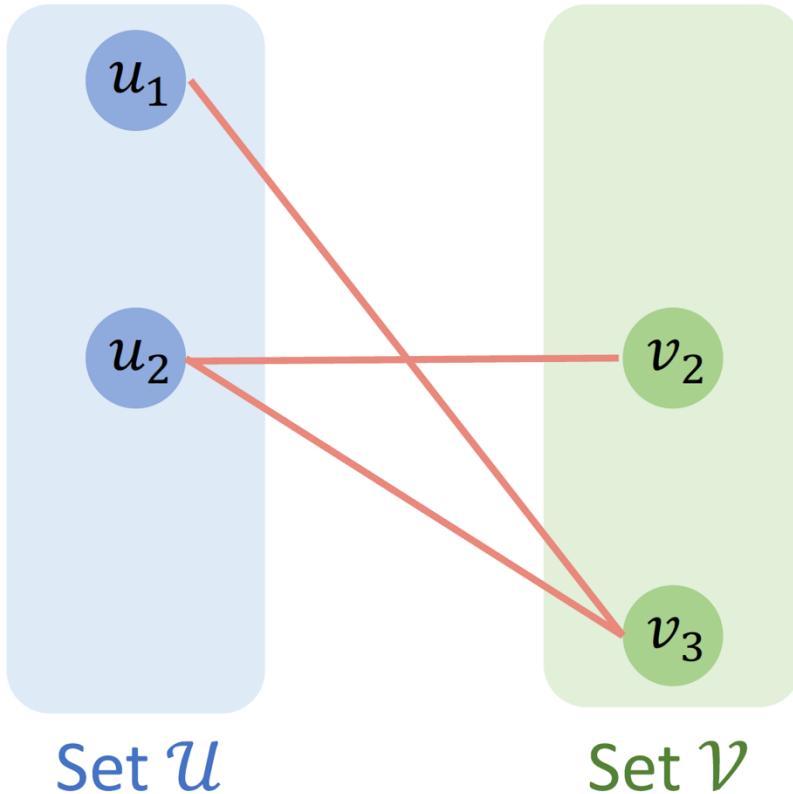
Output the matching



	v_1	v_2	v_3
u_1	0	15	0
u_2	17	0	0
u_3	0	24	86

Minimum-Weight Bipartite Matching

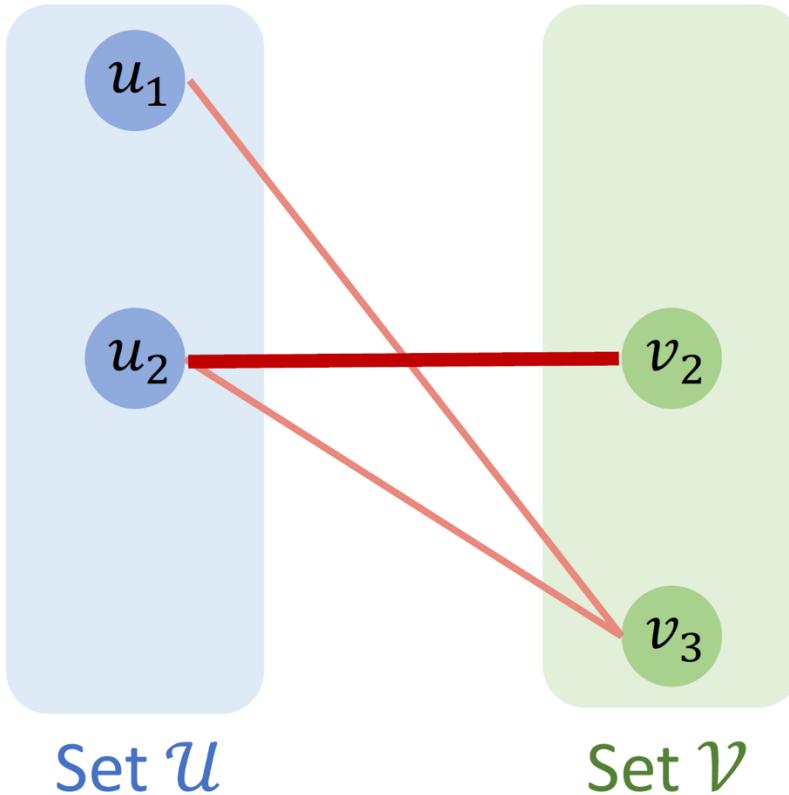
Output the matching



	v_1	v_2	v_3
u_1	0	15	0
u_2	17	0	0
u_3	0	24	86

Minimum-Weight Bipartite Matching

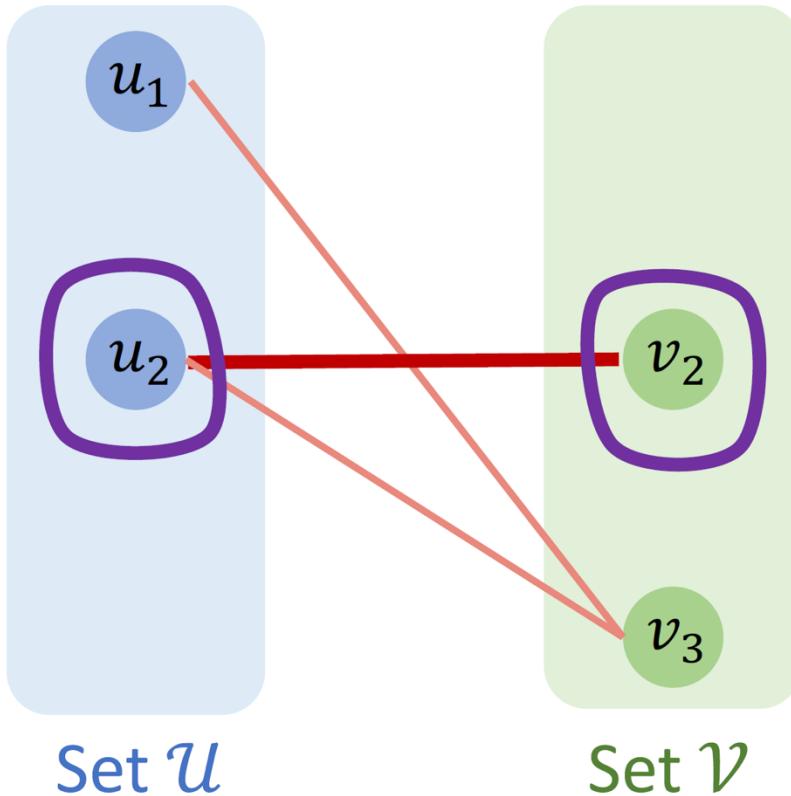
Output the matching



	v_1	v_2	v_3
u_1	0	15	0
u_2	17	0	0
u_3	0	24	86

Minimum-Weight Bipartite Matching

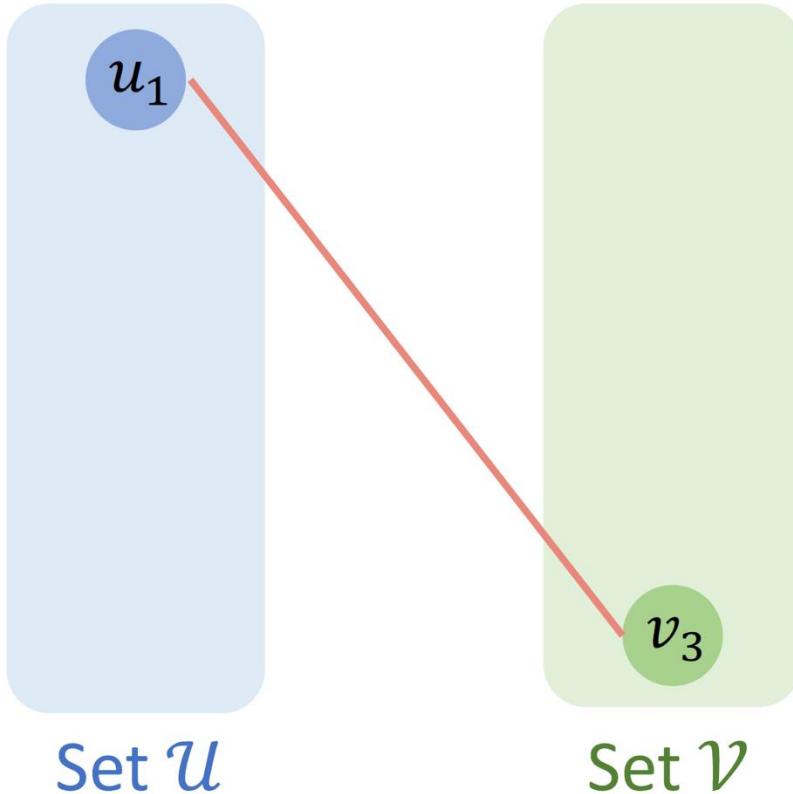
Output the matching



	v_1	v_2	v_3
u_1	0	15	0
u_2	17	0	0
u_3	0	24	86

Minimum-Weight Bipartite Matching

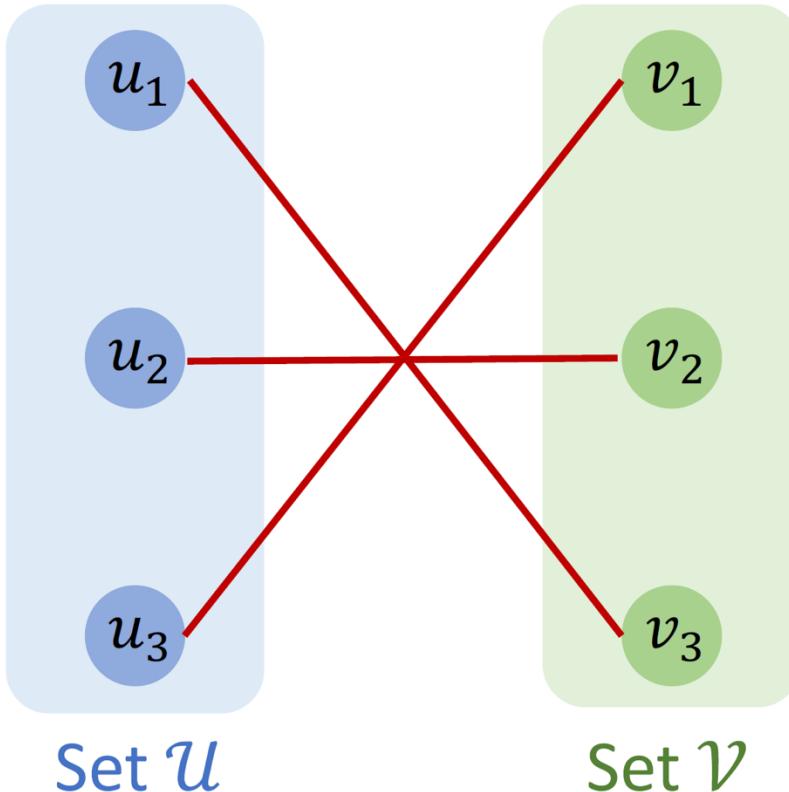
Output the matching



	v_1	v_2	v_3
u_1	0	15	0
u_2	17	0	0
u_3	0	24	86

Minimum-Weight Bipartite Matching

Output the matching

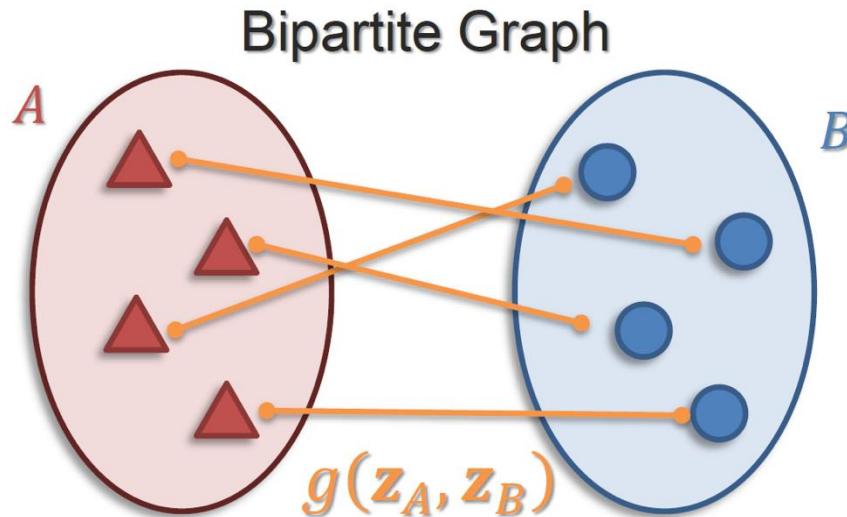


	v_1	v_2	v_3
u_1	0	15	0
u_2	17	0	0
u_3	0	24	86

The matching is

$$\mathcal{S} = \{(u_3, v_1), (u_1, v_3), (u_2, v_2)\}.$$

Global Alignment



Assignment:
(vector of indices)

$$f: A \rightarrow B$$

Similarity weights:

$$w_{i,f(i)} = g(\mathbf{z}_A^i, \mathbf{z}_B^{f(i)})$$

Maximize:

$$\max_{f \in S_n} \sum_{i=1}^N w_{i,f(i)}$$

Initial assumptions:

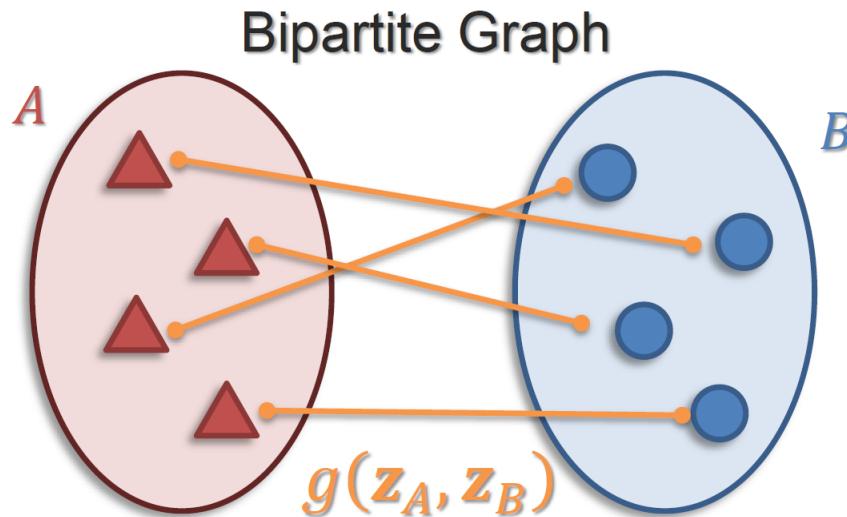
- Same number of elements in A and B modalities
- 1-to-1 “hard” alignment between elements
- All elements assigned (aka “perfect matching”)

→ How to solve?

Naive solution: check all assignments

Hungarian algorithm
Why it works?
Complexity?

Global Alignment



Assignment:
(vector of indices)

$$\underline{f: A \rightarrow B}$$

Similarity weights:

$$\underline{w_{(i,f(i))} = g(\mathbf{z}_A^i, \mathbf{z}_B^{f(i)})}$$

Maximize:

$$\underline{\max_{f \in \text{Perm}(N)} \sum_{i=1}^N w_{i,f(i)}}$$

Initial assumptions:

- Same number of elements in A and B modalities
- 1-to-1 “hard” alignment between elements
- All elements assigned (aka “perfect matching”)

→ How to solve?

Naive solution: check all assignments

Better solution: Linear Programming

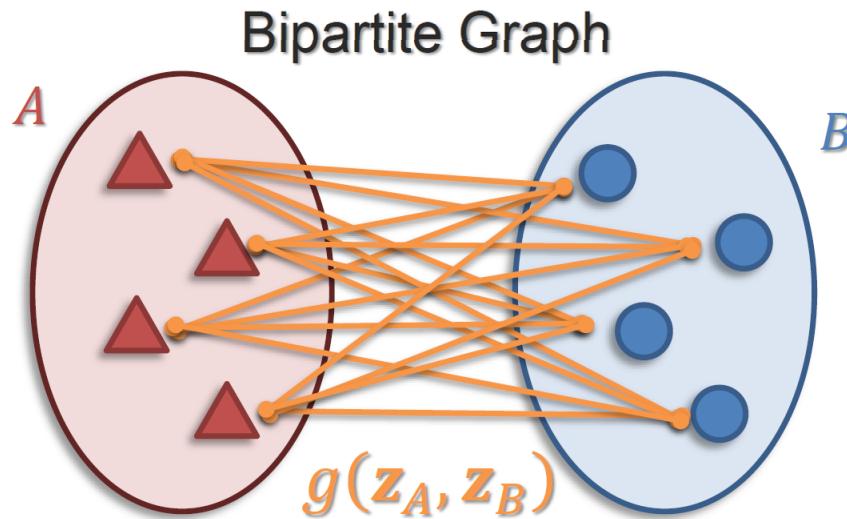
$x_{ij} = 1$ when matching connection, otherwise 0

$$w_{(i,j)} = g(\mathbf{z}_A^i, \mathbf{z}_B^j)$$

$$\max_{\{x_{ij}\}} \sum_{(i,j) \in A \times B} w_{i,j} x_{ij}$$

→ Can be solved with
simplex algorithm

Global Alignment



New assumptions:

- Different number of elements in A and B modalities
- Many-to-many “soft” alignment between elements

→ It can be seen as “transporting” elements from modality A to modality B (and vice-versa)

Assignments: $x_{(i,j)}$: soft alignment between \mathbf{z}_A^i and \mathbf{z}_B^j

Similarity weights: $w_{(i,j)} = g(\mathbf{z}_A^i, \mathbf{z}_B^j)$

Maximize:
$$\max_{\{x_{ij}\}} \sum_{(i,j) \in A \times B} w_{i,j} x_{ij}$$

→ Wassertein distance give optimal transport

内容提纲

① Discrete alignment

① Local alignment

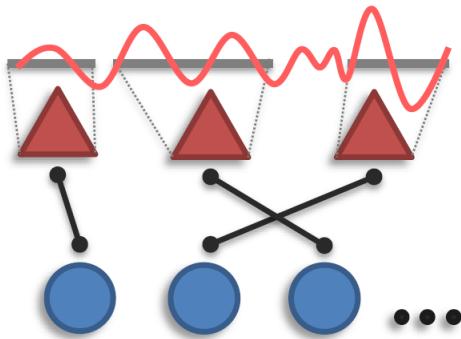
② Global alignment

② Continuous alignment

① Continuous warping

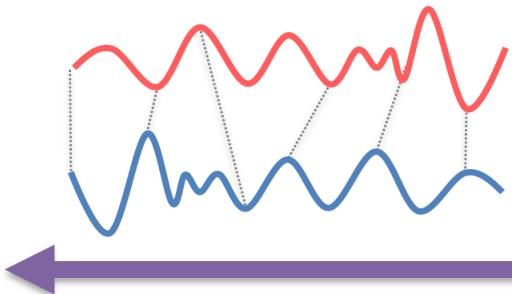
② Discretization and segmentation

Challenge 2b: Continuous Alignment

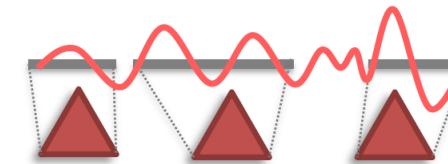


Definition: Model alignment between modalities with continuous signals and no explicit elements

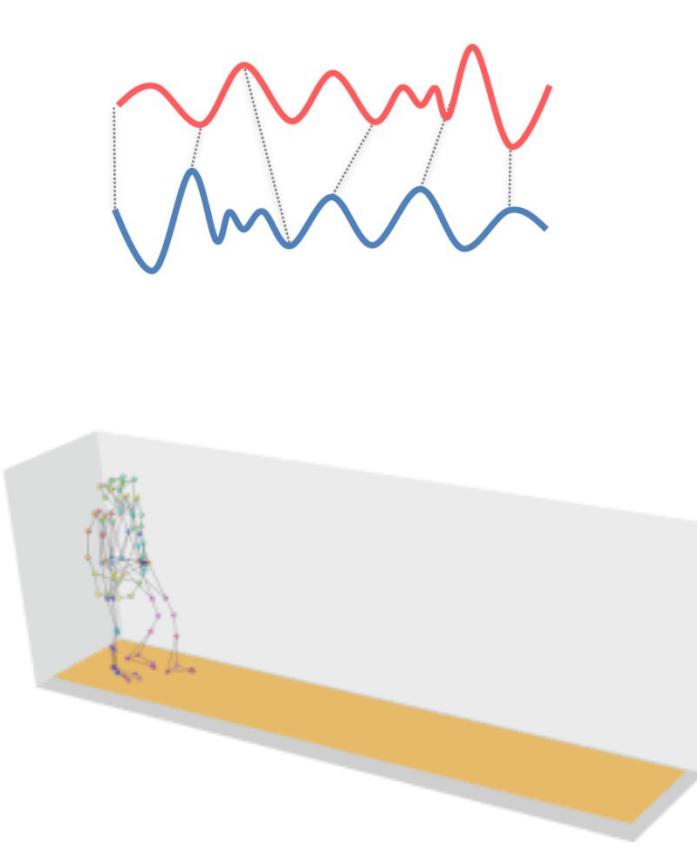
Continuous
warping



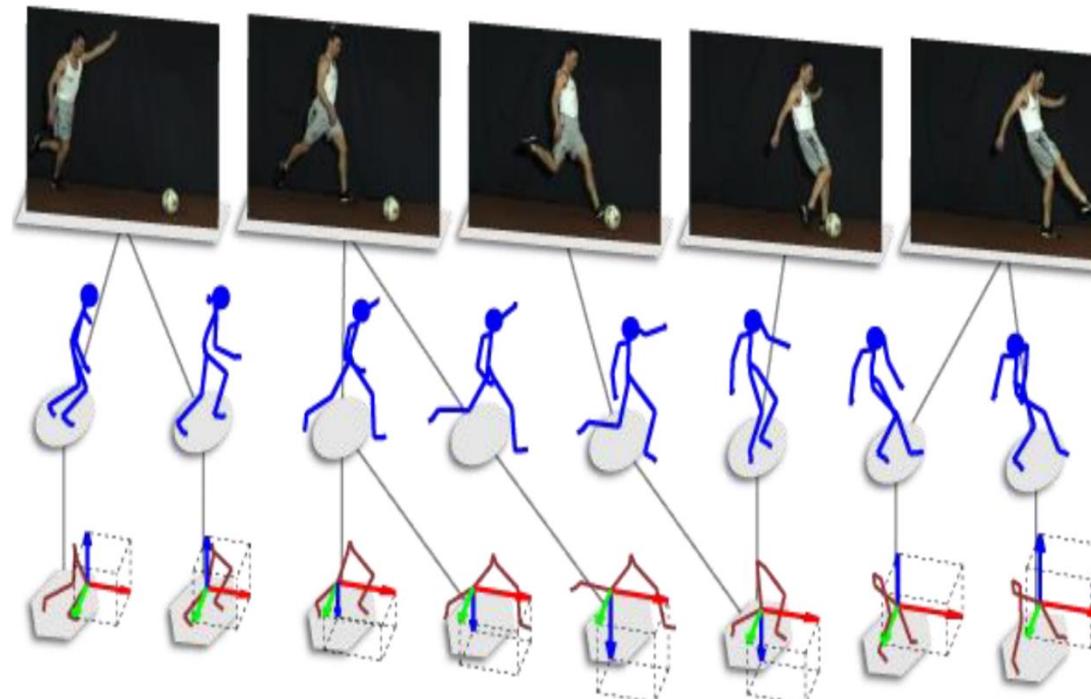
Discretization
(segmentation)



Continuous Warping – Example



→ Aligning video sequences



Dynamic Time Warping (DTW)

We have two unaligned temporal unimodal signals

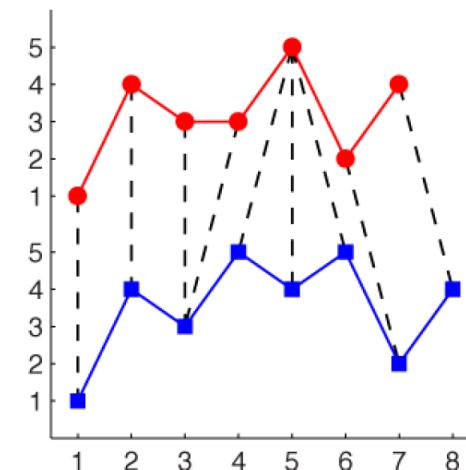
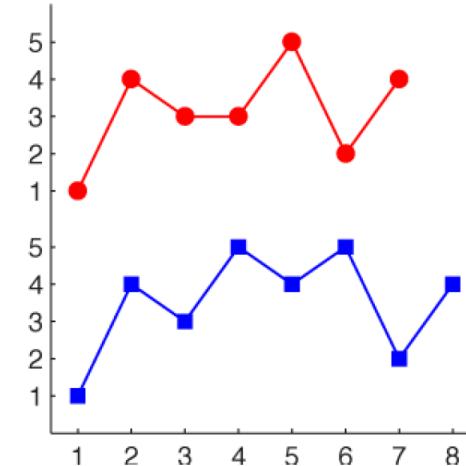
- $\mathbf{X} = [x_1, x_2, \dots, x_{n_x}] \in \mathbb{R}^{d \times n_x}$
- $\mathbf{Y} = [y_1, y_2, \dots, y_{n_y}] \in \mathbb{R}^{d \times n_y}$

Find set of indices to minimize the alignment difference:

$$L(\mathbf{p}^x, \mathbf{p}^y) = \sum_{t=1}^l \left\| x_{p_t^x} - y_{p_t^y} \right\|_2^2$$

where \mathbf{p}^x and \mathbf{p}^y are index vectors of same length

Dynamic Time Warping is designed to find these index vectors!

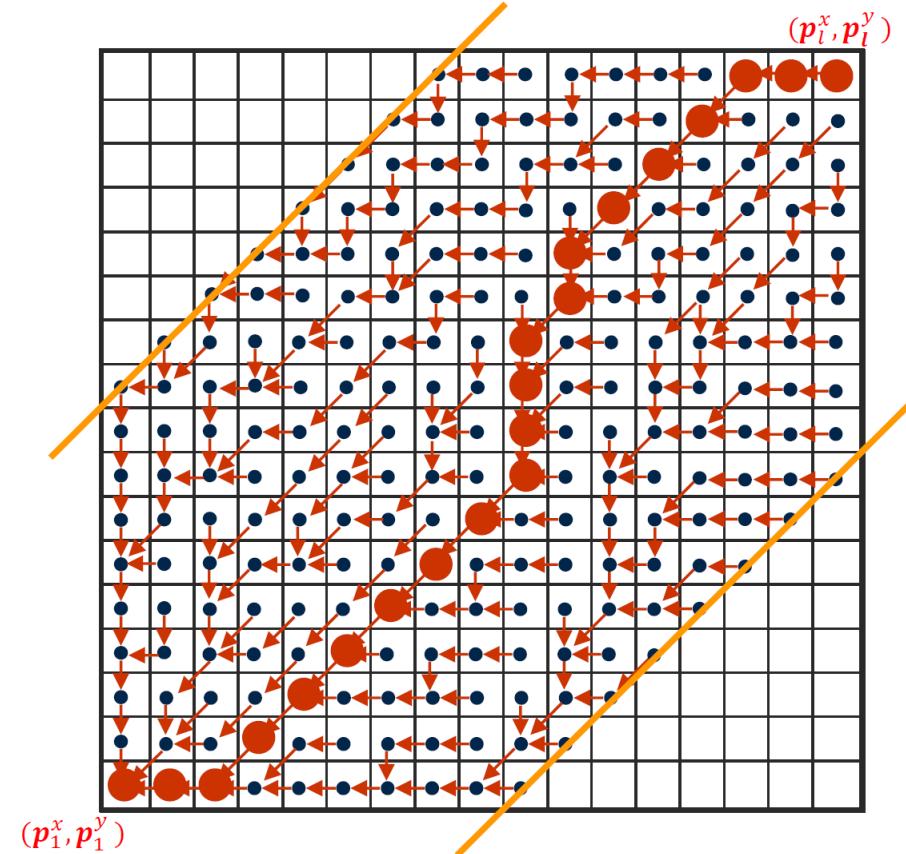


Dynamic Time Warping (DTW)

Lowest cost path in a cost matrix

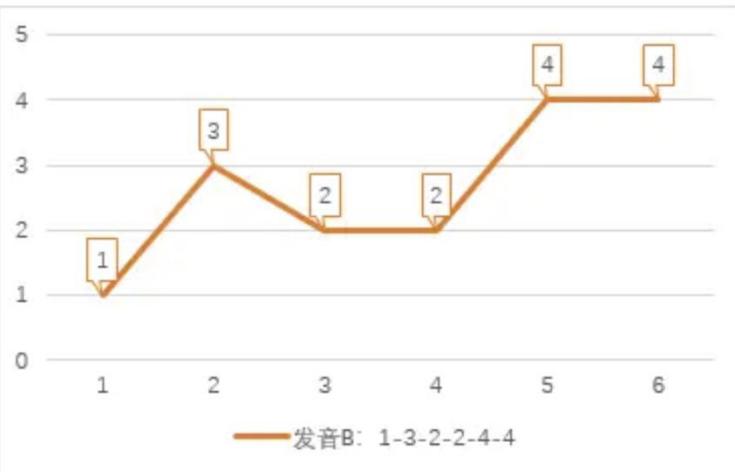
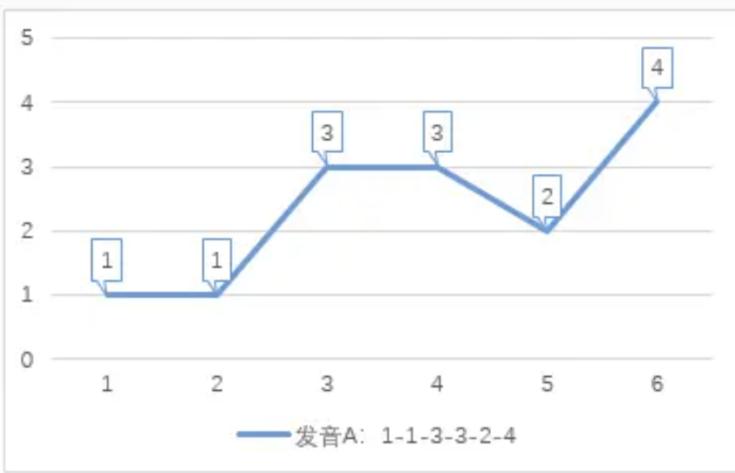
- Restrictions?
 - Monotonicity – no going back in time
 - Continuity - no gaps
 - Boundary conditions - start and end at the same points
 - Warping window - don't get too far from diagonal
 - Slope constraint – do not insert or skip too much

Solved using dynamic programming while respecting the restrictions



Dynamic Time Warping (DTW)

现在有两个人说这个单词，一个人在前半部分拖长，其发音为1-1-3-3-2-4；另一个人在后半部分拖长，其发音为1-3-2-2-4-4



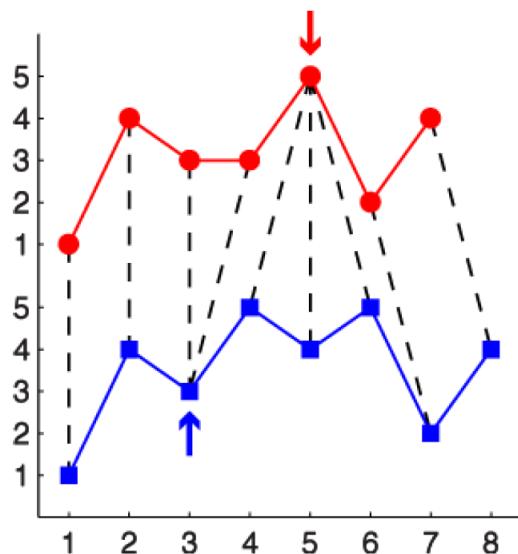
	A(1)=1	A(2) =1	A(3) =3	A(4) =3	A(5) =2	A(6) =4
B(1) =1	0 → 0	2	2	1	3	
B(2) =3	2	2	0 → 0	1	1	
B(3) =2	1	1	1	1	0	2
B(4) =2	1	1	1	1	0	2
B(5) =4	3	3	1	1	2	0
B(6) =4	3	3	1	1	2	0

1. 计算两个序列各个点之间的
距离矩阵
2. 寻找一条从矩阵左上角到右
下角的路径，使得路径上的元
素和最小

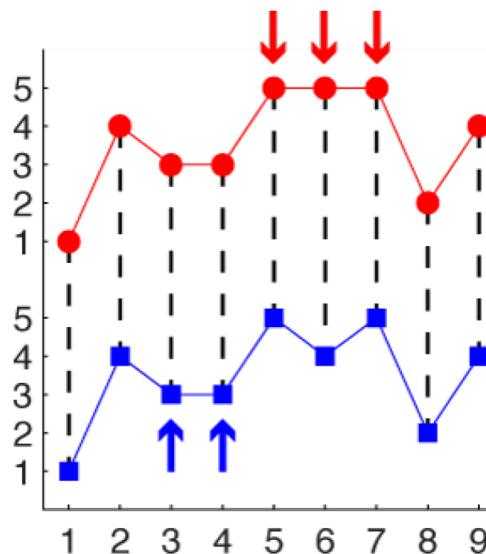
DTW alternative formulation

$$L(\mathbf{p}^x, \mathbf{p}^y) = \sum_{t=1}^{\ell} \|x_{p_t^x} - y_{p_t^y}\|_2^2$$

Replication doesn't change the objective!



=



$$\begin{aligned} &= \mathbf{X} \\ &= \mathbf{Y} \end{aligned}$$

1	0	0	0	0	0	0	0	0
2	0	1	0	0	0	0	0	0
3	0	0	1	0	0	0	0	0
4	0	0	0	1	0	0	0	0
5	0	0	0	0	1	1	1	0
6	0	0	0	0	0	0	0	1
7	0	0	0	0	0	0	0	0
1	2	3	4	5	6	7	8	9

1	0	0	0	0	0	0	0	0
2	0	1	0	0	0	0	0	0
3	0	0	1	1	0	0	0	0
4	0	0	0	0	1	0	0	0
5	0	0	0	0	0	1	0	0
6	0	0	0	0	0	0	1	0
7	0	0	0	0	0	0	0	1
1	2	3	4	5	6	7	8	9

Alternative objective:

$$L(\mathbf{W}_x, \mathbf{W}_y) = \|\mathbf{X}\mathbf{W}_x - \mathbf{Y}\mathbf{W}_y\|_F^2$$

Frobenius norm $\|A\|_F^2 = \sum_i \sum_j |a_{i,j}|^2$



\mathbf{X}, \mathbf{Y} – original signals (same #rows, possibly different #columns)

$\mathbf{W}_x, \mathbf{W}_y$ - alignment matrices

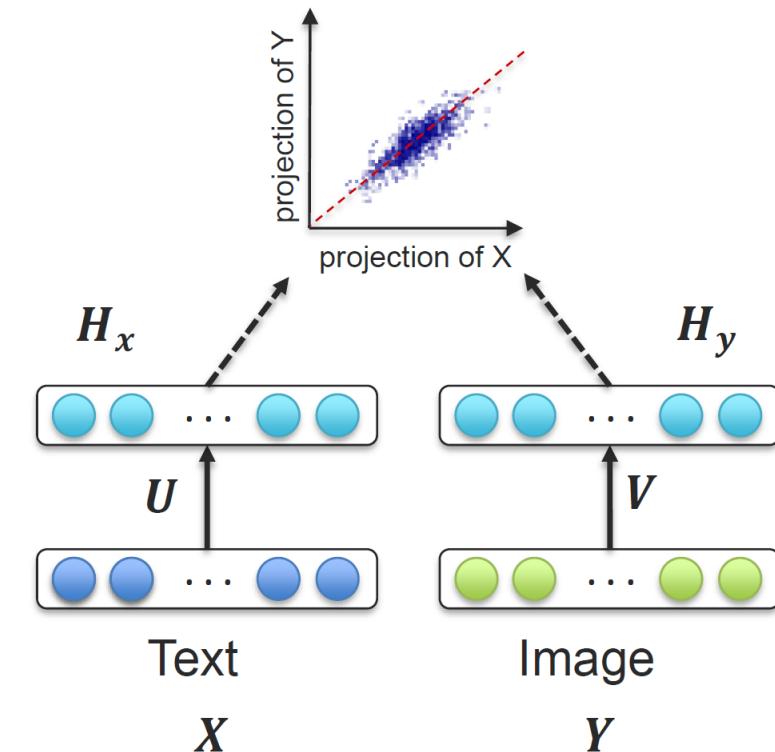
Canonical Correlation Analysis – Reminder

CCA loss can also be re-written as:

$$L(\mathbf{U}, \mathbf{V}) = \|\mathbf{U}^T \mathbf{X} - \mathbf{V}^T \mathbf{Y}\|_F^2$$

subject to:

$$\mathbf{U}^T \Sigma_{YY} \mathbf{U} = \mathbf{V}^T \Sigma_{YY} \mathbf{V} = \mathbf{I}, \mathbf{u}_{(j)}^T \Sigma_{XY} \mathbf{v}_{(i)} = 0$$



Canonical Time Warping

Dynamic Time Warping + Canonical Correlation Analysis = Canonical Time Warping

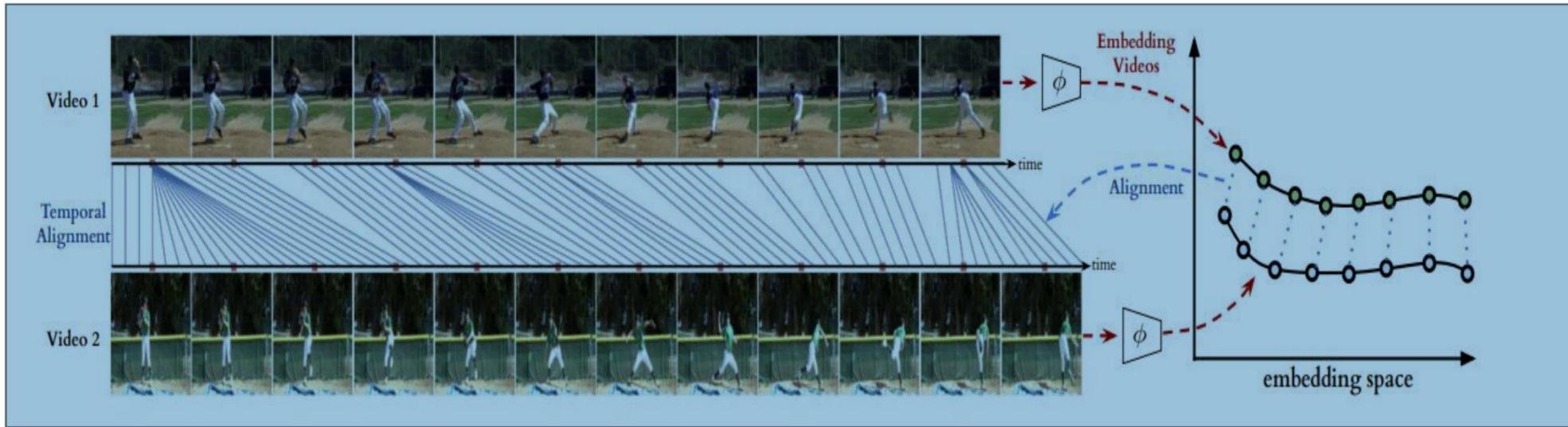
$$L(\mathbf{U}, \mathbf{V}, \mathbf{W}_x, \mathbf{W}_y) = \left\| \mathbf{U}^T \mathbf{X} \mathbf{W}_x - \mathbf{V}^T \mathbf{Y} \mathbf{W}_y \right\|_F^2$$

Allows to align multi-modal or multi-view (same modality but from a different point of view)

- $\mathbf{W}_x, \mathbf{W}_y$ – temporal alignment
- \mathbf{U}, \mathbf{V} – cross-modal (spatial) alignment

Temporal Alignment and Neural Representation Learning

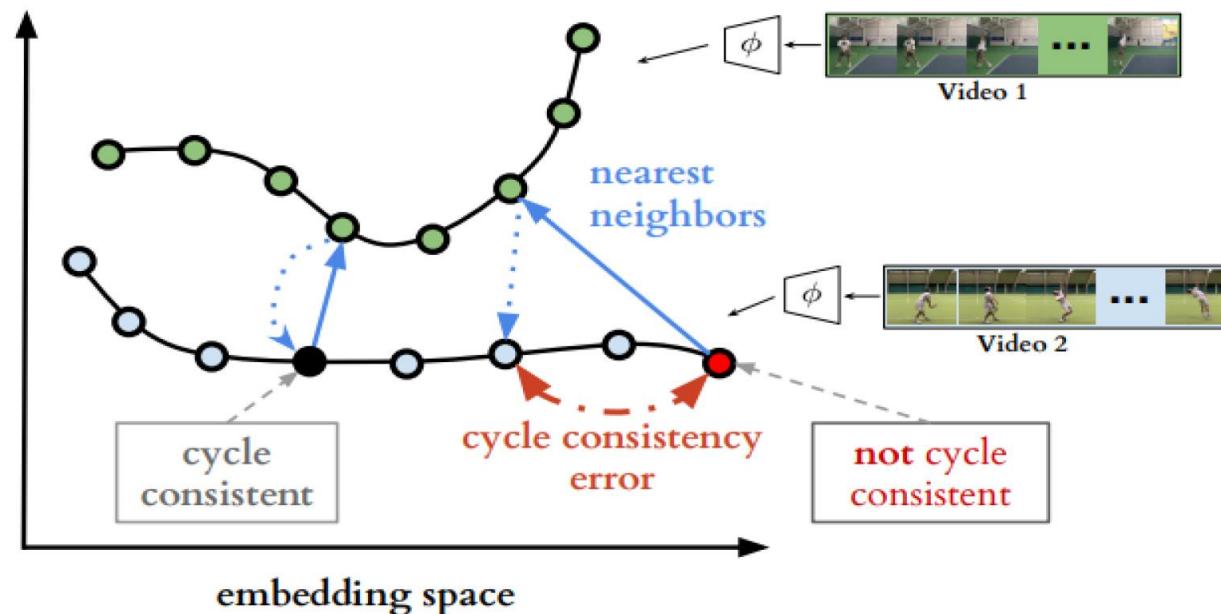
Premise: we have paired video sequences that can be temporally aligned



How can we define a loss function to enforce the alignment between sequences while at the same time learning good representations?

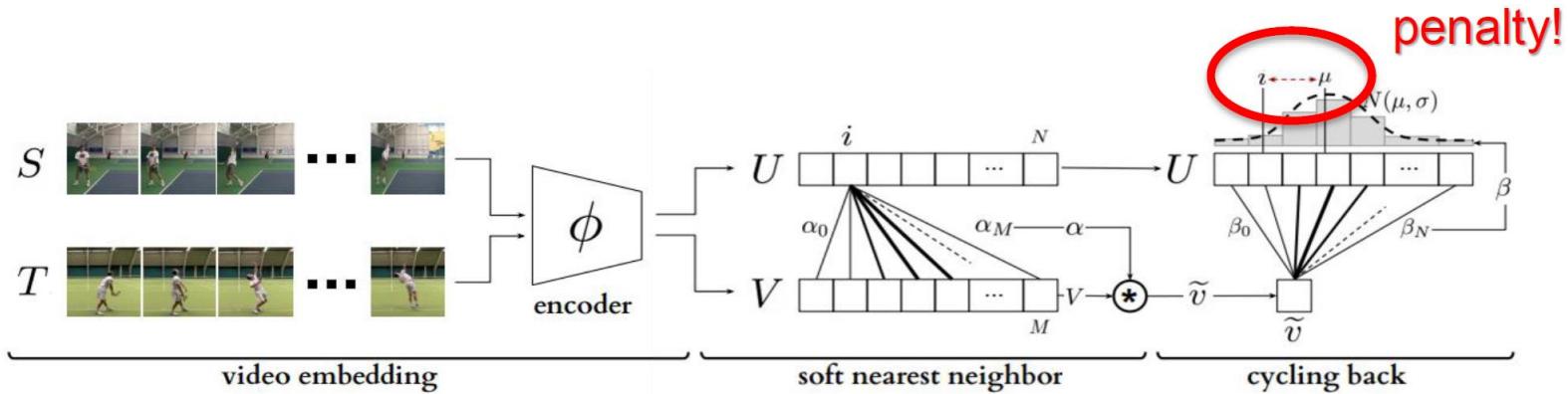
Temporal Cycle-Consistency Learning

Solution: Representation learning by enforcing **Cycle consistency**



Main idea: My closest neighbor also views me as their closest neighbor

Temporal Cycle-Consistency Learning



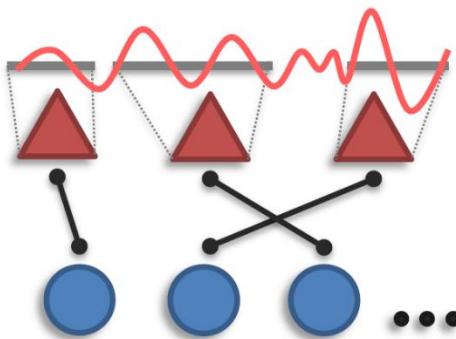
Compute “soft” / “weighted” nearest neighbour:

$$\text{distances: } \alpha_j = \frac{e^{-\|u_i - v_j\|^2}}{\sum_k^M e^{-\|u_i - v_k\|^2}} \quad \text{Soft nearest neighbor: } \tilde{v} = \sum_j^M \alpha_j v_j,$$

Find the nearest neighbor the other way and then penalize the distance:

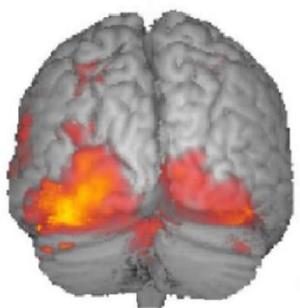
$$\beta_k = \frac{e^{-\|\tilde{v} - u_k\|^2}}{\sum_j^N e^{-\|\tilde{v} - u_j\|^2}} \quad L_{cbr} = \frac{|i - \mu|^2}{\sigma^2} + \lambda \log(\sigma)$$

Discretization (aka Segmentation)

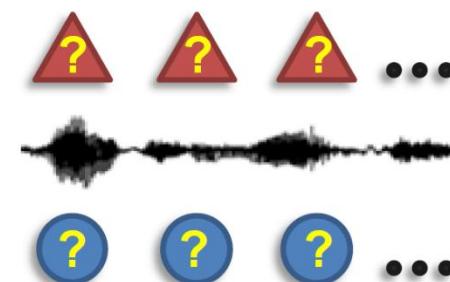
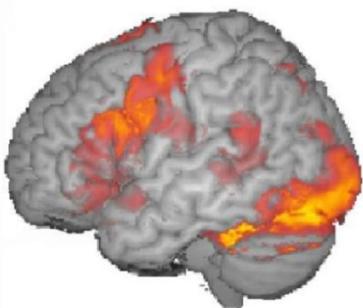


Common assumptions: ① Segmented elements

Examples:



Medical imaging



Signals

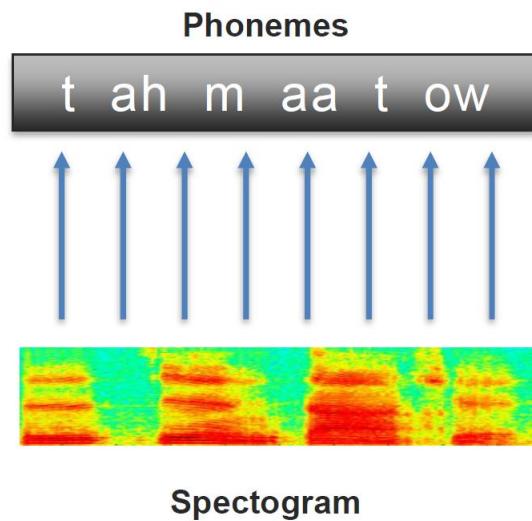


Images

objects

Discretization – Example

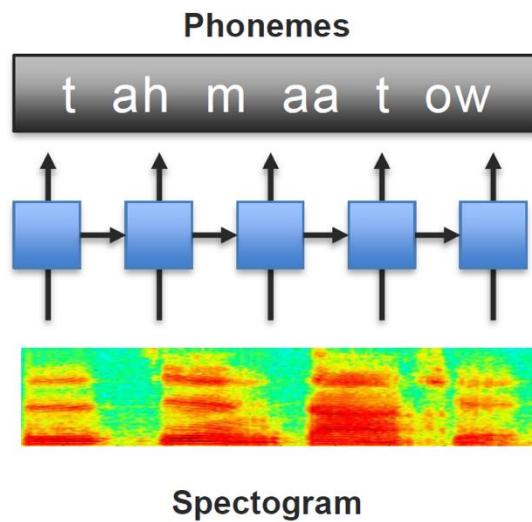
Sequence Labeling and Alignment



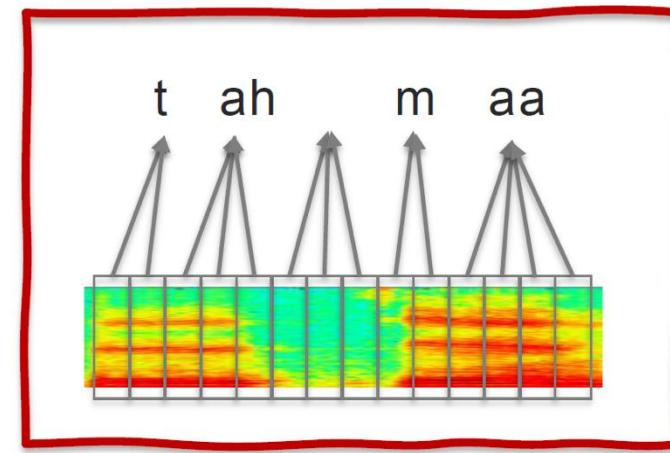
How can we predict the sequence
of phoneme labels?

Low-rank Fusion with Trimodal Input

Sequence Labeling and Alignment



Challenge: many-to-1 alignment



How can we predict the sequence
of phoneme labels?

Discretization – A Classification Approach

Connectionist Temporal Classification

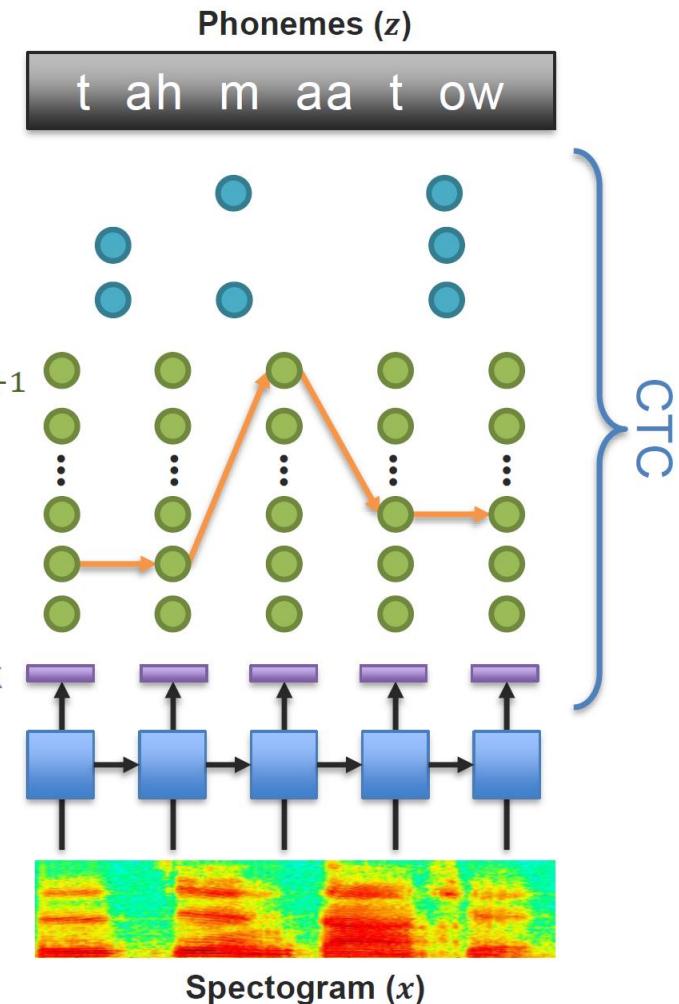
④ Most probable sequence labels

③ Predicted labels l

for 'blank' or no label

② Path π over the activations:

① Output activations (distribution):



Discretization and Representation – Cluster-based Approaches

HUBERT: Hidden-Unit BERT

